

Heavy neutrino dark matter in the solar system

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ABSTRACT

We study a simple model of dark matter that is gravitationally clustered around the sun in the form of a spherical halo of a degenerate gas of heavy neutrinos. It is shown that for neutrino masses $m_\nu \lesssim 16 \text{ keV}/c^2$, the resulting matter distribution is consistent with the constraints on the mass excesses within the orbits of the outer planets, as obtained from astrometrical and the Pioneer 10/11 and Voyager 1/2 (Anderson et al. 1995) ranging data. However, the anomalous acceleration recently detected in the Pioneer 10/11 data that is approximately constant between 40 AU and 60 AU (Anderson et al. 1998; Turyshev et al. 1999) is incompatible with both our model and earlier Pioneer 10/11 ranging data for the outer planets. We then calculate the planetary and asteroidal perihelion shifts generated by such a neutrino halo. For $m_\nu \lesssim 16 \text{ keV}/c^2$, our results are consistent with the observational data on Mercury, Venus, Earth and Icarus. Finally, we propose to detect this neutrino halo directly with a dedicated experiment on Earth, observing the X-rays emitted in the radiative decay of the heavy neutrino into a light neutrino and a photon.

Subject headings: Gravitation-Dark matter-Solar system:general

1. Introduction

It is well known that the properties of galactic systems pose a great challenge to gravity theories. For virtually all spiral galaxies the galactic rotation curves tend towards some constant value for large distances from the center of the galaxy. This is clearly either inconsistent with the mass distribution inferred from the distribution of visible stars, or, even more challenging, in contradiction with the laws of Newtonian dynamics.

Most widely accepted is the conservative explanation of this discrepancy in terms of dark matter (DM) (e.g. Ashman 1992) which presumes that the visible stars are embedded in a massive, nearly spherical halo of nonluminous matter. The mass of the halo varies from one galaxy to another, but in general it constitutes 90% of the total mass (Tremaine 1992). While the DM hypothesis could explain the flat rotation curves of the galaxies in a consistent manner, it has its own troubles, in particular: (i) there is no compelling model for the formation of the DM halo, and (ii) despite much effort, so far no known form of matter lends itself to a satisfactory understanding of the DM halo. For instance, the microlensing experiments MACHO (Alcock et al. 1996, 1997) and EROS (Ansari et al. 1996), are as yet far from explaining the galactic DM halo in terms of massive astrophysical compact halo objects (MACHOs). In fact, it is by no means clear

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that the DM halo should be made of astrophysical low-mass black holes, brown dwarfs, old white dwarfs or more exotic nonbaryonic MACHOs. On the contrary, the DM halo may very well consist of loose clouds of weakly interacting (i.e. nonbaryonic) massive particles (WIMPs) such as SUSY neutralinos, neutrinos or axions (Jungman et al. 1996). Microlensing experiments have ruled out a large class of possible compact baryonic DM components. As gaseous baryonic components are also largely excluded, it has been argued that at least some nonbaryonic DM is required to explain the DM halo in our Galaxy (Freese et al. 1999). Of course, if DM is in the form of nonbaryonic particle clouds, it will be everywhere in our Galaxy and it may thus also be observed in the solar system.

An alternative explanation for the galactic rotation curves is based on the possibility that Newton’s gravitational law ceases to be valid at small accelerations (Milgrom 1983; Bekenstein 1992) or large scales (Sanders 1990). This challenging hypothesis has been discussed in a number of papers (e.g. Eckhardt 1993; Hammond 1994). For instance, it has been shown (Sanders 1986; Nieto and Goldman 1992) that a modified gravitational potential of the form

$$\phi = \frac{-GM}{r(1 + \alpha)} \left[1 + \alpha e^{-r/r_0} \right],$$

with $\alpha = -0.9$ and $r_0 \approx 30$ kpc, could indeed explain the flat rotational curves for most of the galaxies. In contrast to the DM hypothesis, there would be no detectable corrections to Newton’s gravity, as the exponential term is unity with high accuracy for the solar system. We would thus recover the standard form for the gravitational potential $\phi = -GM/r$ within the solar system. The problem with this hypothesis is that it is not clear how to incorporate the modified gravitational potential in a fully general relativistic theory.

The main motivation for studying DM in the solar system is that galaxies could have halos consisting of weakly interacting massive particles (WIMPs) (Trimble 1987). Indeed, if DM exists in the form of WIMPs, one would expect this exotic form of DM to penetrate both the galactic disk and the solar system. The determination of the total matter density in the vicinity of the sun is a classical problem of astrophysics (see Tremaine (1990) and Oort (1965) for a comprehensive review of early research). It allows for a significant amount of DM that could have condensed into a DM halos around the stars. However, the conditions on particle masses and interaction cross sections under which this condensation would have taken place during star formation, have not been worked out in detail. In fact, it is difficult to envisage a gravitational interaction mechanism that would lead to capture of significant amounts of weakly interacting DM particles. Assuming these particles to be dissipationless, they would be part of the galactic halo and move with the velocities of the order of $10^{-3}c$. In this case very few DM particles could be trapped in orbits around the sun and the DM density would be nearly constant near the sun. However, Bahcall (1984a, 1984b) has argued for the possible existence of a galactic disk component of nonluminous matter, not necessarily weakly interacting, with a density of the order of luminous disk matter, i.e. $0.1 M_\odot \text{pc}^{-3}$. Such DM could have been trapped around the sun since its formation. In fact, it is possible that “nonequilibrium” gravitational interaction between a cloud of weakly interacting DM particles and a molecular cloud, could play an important role in the formation of stars with DM halos around them. The obvious place to test such a hypothesis is the solar system. In fact, Mikkelsen and Newman (1977) established a bound of $M_d \lesssim 7 \times 10^{-8} M_\odot$ for DM within the Earth’s orbit, while Khloper et al. (1991) made estimates of 10^{-7} to $10^{-6} M_\odot$ of shadow matter being captured by a baryonic matter star. If a DM halo exists around the sun, it would induce an additional perihelion precession of the planetary orbits (Braginsky et al. 1992). More recently, Anderson et al. (1995) have investigated the bounds on DM within the orbits of different outer planets using the data from Pioneer 10/11 and Voyager 1/2. They found, within Uranus’ orbit, an upper bound for the DM mass of $M_d \lesssim 0.5 \times 10^{-6} M_\odot$, while a bound of $M_d \lesssim 3 \times 10^{-6} M_\odot$

was obtained within Neptune’s orbit, and finally, within Jupiter’s orbit, the DM mass was determined to $M_d = (0.12 \pm 0.027) \times 10^{-6} M_\odot$.

Thus, in this paper, we assume that massive neutrinos play the role of the weakly interacting DM particles, that have been trapped around the sun during its formation, and we investigate the consequences of the existence of such a degenerate neutrino halo. In fact, the idea that DM is made of massive neutrinos that can cluster around a baryonic star has been developed in a series of papers (Viollier et al. 1992, Viollier 1993; Viollier 1994). This model resembles the Thomas-Fermi model of an atom or ion, where the degenerate electron cloud around the nucleus is described by a mean electrostatic field. The difference between the two models is the sign of the interaction. While the model of the neutrino halo around a star admits also solutions without star at the center, i.e. pure neutrino balls (Viollier 1994), there is no corresponding solution for a nucleus with charge $Z = 0$. Here it is important to note that pure neutrino balls could also describe the supermassive compact dark objects at the galactic centers, e.g. at the center of our Galaxy or M87 (Tsiklauri and Viollier 1998; Bilić, Munyaneza and Viollier 1999; Munyaneza, Tsiklauri and Viollier 1998, 1999; Munyaneza and Viollier 1999).

The purpose of this paper is to study the mass distribution of degenerate neutrino DM around the sun and to compare it with the most recent observational data from Pioneer 10/11 and Voyager 1/2 (Anderson et al. 1995). We will also investigate the effect of a possible neutrino halo on the precession of the perihelion of planetary or asteroidal orbits.

This paper is organized as follows: In section 2, we establish the basic equations of our model and study the mass distribution of the neutrino halo around the sun. In section 3, we investigate the planetary and asteroidal perihelion shifts due to a neutrino halo. We discuss other observational implications of such a neutrino halo in section 4 and summarize our results with a discussion in section 5.

2. A degenerate heavy neutrino halo around the sun

Let us characterize the spherically symmetric cloud of degenerate heavy neutrino matter around a baryonic star by its gravitational potential $\Phi(r)$, pressure $P_\nu(r)$, and mass density $\rho_\nu(r)$. In nonrelativistic approximation, these three quantities are linked through Poisson’s equation

$$\Delta\Phi = 4\pi G\rho_\nu, \quad (1)$$

the condition for hydrostatic equilibrium between the gravitational and degeneracy pressures of heavy neutrino matter

$$\frac{dP_\nu}{dr} = -\rho_\nu \frac{d\Phi(r)}{dr}, \quad (2)$$

and the polytropic equation of state of a degenerate nonrelativistic Fermi gas

$$P_\nu = K\rho_\nu^{5/3}. \quad (3)$$

Here the constant K is given by

$$K = \left(\frac{6}{g_\nu}\right)^{2/3} \frac{\pi^{4/3} \hbar^2}{5m_\nu^{8/3}}, \quad (4)$$

g_ν being the spin degeneracy factor of the neutrinos and antineutrinos, i.e. $g_\nu = 2$ for Majorana neutrinos and $g_\nu = 4$ for Dirac neutrinos and antineutrinos. The pressure and density vanish at the radius R_0 of the neutrino halo, i.e.

$$\rho_\nu(R_0) = P_\nu(R_0) = 0. \quad (5)$$

Outside the neutrino halo, $r > R_0$, we thus recover the standard solution of Poisson's equation (1)

$$\Phi(r) = -\frac{G(M_\nu + M_B)}{r}, \quad (6)$$

where M_ν is the mass of the neutrino halo, i.e.

$$M_\nu = \int_0^{R_0} 4\pi\rho_\nu(r)r^2 dr, \quad (7)$$

and M_B is the mass of the pointlike baryonic star at the center of the neutrino halo.

Introducing the dimensionless potential variable v and radius x , by

$$v(x) = \frac{r}{GM_\odot} \left(\Phi(R_0) - \Phi(r) \right), \quad (8)$$

$$x = \frac{r}{a_\nu}, \quad (9)$$

where a_ν is an appropriate length scale

$$a_\nu = \left(\frac{3\pi\hbar^3}{4\sqrt{2}m_\nu^4 g_\nu G^{3/2} M_\odot^{1/2}} \right)^{2/3} = 0.4129 \text{ pc} \left(\frac{17.2 \text{ keV}}{m_\nu c^2} \right)^{8/3} \left(\frac{2}{g_\nu} \right)^{2/3}, \quad (10)$$

we arrive at the nonlinear Lane-Emden differential equation (Viollier, Leimgruber and Trautmann 1992, Viollier 1994) with index $n = 3/2$

$$\frac{d^2 v}{dx^2} = \frac{-v^{3/2}}{x^{1/2}}, \quad (11)$$

subject to the boundary conditions

$$v(0) = \frac{M_B}{M_\odot} \quad \text{and} \quad v(x_0) = 0, \quad (12)$$

where $x_0 = R_0/a_\nu$ is the radius of the halo in units of a_ν . The first condition in (12) arises from the fact that, near the origin, the potential energy is dominated by M_B . Thus $M_B = 0$ corresponds to a pure neutrino ball without a pointlike source at the center. The properties of such neutrino balls have been discussed in a number of papers (Tsiklauri and Viollier 1998; Munyaneza, Tsiklauri and Viollier 1998, 1999; Munyaneza and Viollier 1999). In particular, it has been shown that a neutrino ball of $M \approx 2.6 \times 10^6 M_\odot$ is a viable alternative to the supermassive black hole that many believe to exist at the Galactic center, as (i) it is consistent with the upper limit of the size of the supermassive compact dark object as determined by the motion of stars near Sgr A* (Munyaneza, Tsiklauri and Viollier 1998, 1999), and (ii) the bulk part of its infrared to radiowave spectrum, up to wavelengths of $\lambda = 0.3 \text{ cm}$, can be described by standard thin accretion disk theory (Bilić and Viollier 1998; Bilić, Tsiklauri and Viollier 1998; Tsiklauri and Viollier 1999; Munyaneza and Viollier 1999).

The neutrino matter density may be expressed in terms of the potential variable v as

$$\begin{aligned} \rho_\nu(r) &= \frac{8m_\nu^8 g_\nu^2 G^3 M_\odot^2}{9\pi^3 \hbar^6} \left(\frac{v}{x} \right)^{3/2} \\ &= 1.12912 M_\odot \text{pc}^{-3} \left(\frac{m_\nu c^2}{17.2 \text{ keV}} \right)^8 \left(\frac{g_\nu}{2} \right)^2 \left(\frac{v}{x} \right)^{3/2}. \end{aligned} \quad (13)$$

Using Eqs. (7) and (13), the neutrino matter mass enclosed within a radius r from the sun can be written as

$$\mathcal{M}_\nu(r) = -M_\odot \left(xv'(x) - v(x) + v(0) \right). \quad (14)$$

with

$$\mathcal{M}_\nu(R_0) = -M_\odot \left(x_0 v'(x_0) + v(0) \right) = M_\nu. \quad (15)$$

Of course, the mass density ρ_ν diverges at the origin as $x^{-3/2}$ due to the pointlike nature of the baryonic source. However, the integral in Eq. (7) converges, yielding a finite total mass of the neutrino halo M_ν . In Fig. 1 we plot the mass $\mathcal{M}_\nu(r)$ that is enclosed within a radius r from the sun, for various values of the neutrino mass m_ν . The slope of v at the center has been fixed to $v'(0) = 1$, yielding a total mass of the neutrino halo of $M_\nu = 0.7 M_\odot$. Of course, by varying $v'(0)$, the halo could have any mass. Also shown in Fig. 1 is the bound on DM mass within the Earth's orbit, i.e. $M_d \lesssim 7 \times 10^{-8} M_\odot$ (Mikkelsen and Newman 1977) as well as the constraints on the mass excesses within the orbits of various outer planets, as obtained from astrometrical and the Voyager 1 and 2 and Pioneer 10 and 11 ranging data (Anderson et al. 1995). According to these data the DM mass contained within Jupiter's orbit is $M_d = (0.12 \pm 0.027) \times 10^{-6} M_\odot$, within Uranus' orbit $M_d \lesssim 0.5 \times 10^{-6} M_\odot$, and within Neptune's orbit $M_d \lesssim 3 \times 10^{-6} M_\odot$. A DM mass of a few $10^{-4} M_\odot$ over the range from 40 AU to 60 AU as shown in Fig. 1 would be consistent with the anomalous acceleration of $a_P \approx 7.5 \times 10^{-8} \text{cm/s}^2$ observed in the Pioneer 10/11, Galileo and Ulysses data (Anderson et al. 1998; Turyshev et al. 1999).

Based on our model, we can determine the neutrino-mass range compatible with the limits on DM for the various planetary orbits. The constraints on the DM mass within Earth's orbit yields the neutrino mass limits

$$\begin{aligned} m_\nu c^2 &\leq 21.8 \text{ keV for } g_\nu = 2 \\ m_\nu c^2 &\leq 18.3 \text{ keV for } g_\nu = 4. \end{aligned} \quad (16)$$

Taking the DM data within Jupiter's orbit at face value and interpreting DM as degenerate neutrino matter, the neutrino mass limits are

$$\begin{aligned} 12.8 \text{ keV} &\leq m_\nu c^2 \leq 14.2 \text{ keV for } g_\nu = 2, \\ 10.8 \text{ keV} &\leq m_\nu c^2 \leq 11.9 \text{ keV for } g_\nu = 4. \end{aligned} \quad (17)$$

However, one should perhaps interpret this range of neutrino masses as a lower limit, as Jupiter tends to eject any matter within its orbit (Anderson et al. 1995).

For DM within Uranus' orbit, the upper limits for m_ν are

$$\begin{aligned} m_\nu c^2 &\leq 12 \text{ keV for } g_\nu = 2, \\ m_\nu c^2 &\leq 10 \text{ keV for } g_\nu = 4, \end{aligned} \quad (18)$$

and finally for Neptune's orbit, the bound on the neutrino mass is

$$\begin{aligned} m_\nu c^2 &\leq 16 \text{ keV for } g_\nu = 2, \\ m_\nu c^2 &\leq 13.5 \text{ keV for } g_\nu = 4. \end{aligned} \quad (19)$$

We now explore what the total mass of the neutrino halo should be, in order to be consistent with the observational data. For this purpose, we have calculated the mass $\mathcal{M}_\nu(r)$ that is enclosed within a radius r , for various total masses M_ν of the halo. The results are shown in Fig. 2, where the neutrino mass is fixed to $m_\nu = 14 \text{ keV}/c^2$. As suggested by this figure, the neutrino halo cannot be heavier than $\sim 10^2 M_\odot$, otherwise it would be inconsistent with the constraints on the observed mass excesses within the orbits of the outer planets. In Fig. 3, the acceleration a_ν due to the presence of a neutrino halo is plotted as a function of the radius. Also shown is the range of allowed values for the acceleration at the positions of various planets (Anderson et al. 1995). We thus conclude that our model is consistent with the observations of the outer planets (Anderson et al. 1995). However, it cannot explain the apparent anomalous weak long-range acceleration seen in the Pioneer 10/11, Galileo, and Ulysses data (Anderson et al. 1998; Turyshev et al. 1999) indicated with a horizontal bar in Fig. 3. These groups claim that there is an anomalous acceleration towards the sun of $a_P \approx 7.5 \times 10^{-8} \text{ cm/s}^2$. No variation of a_P with distance from the sun was found over a range of 40 to 60 AU. Of course, our neutrino halo would also contribute to the anomalous acceleration a_P . Setting $a_P = a_\nu$, our model predicts a decrease of the anomalous acceleration due to the fact that the neutrino halo mass enclosed within a distance r from the sun scales as $r^{3/2}$. However, if the anomalous acceleration is indeed constant, as the recent observations (Anderson et al. 1998; Turyshev et al. 1999) seem to indicate, the enclosed mass should increase like r^2 which is clearly inconsistent with the astrometrical and previous ranging data. Of course, there have been some attempts to explain the Pioneer anomalous acceleration (Katz 1998, Murphy 1998), but it is perhaps fair to say that the jury on the explanation of this real or spurious effect is still out. In Fig. 4, we present the mass M_ν as a function of the radius R_0 of the neutrino halo. It is interesting to note that there is always maximal radius of a halo around a baryonic star. For $M_B = M_\odot$, $m_\nu = 14 \text{ keV}/c^2$ and $g_\nu = 2$, this maximal radius turns out to be $R_{max} \approx 5 \text{ lyr}$, which is the scale of the interstellar distances. The corresponding neutrino halo mass would be $M_{\nu,max} \approx 3M_\odot$.

3. Planetary and asteroidal perihelion shifts

A possible neutrino halo will, of course, affect the perihelion shifts of the planets and asteroids of the solar system. Using Eq. (8), the gravitational potential $\Phi(r)$ of a sun that is immersed in a neutrino halo can be written as

$$\Phi(r) = \Phi_B(r) + \delta\Phi_\nu(r), \quad (20)$$

where $\Phi_B(r)$ is the Newtonian potential due to the sun ($M_B = M_\odot$), i.e.

$$\Phi_B(r) = -\frac{GM_\odot}{r}, \quad (21)$$

and $\delta\Phi_\nu(r)$ is the contribution of the neutrino halo to the potential

$$\delta\Phi_\nu(r) = \frac{GM_\odot}{a_\nu} \left(\frac{1 - v(x)}{x} + v'(x_0) \right). \quad (22)$$

Here v is the potential variable that satisfies the Lané-Emden equation with a pointlike solar mass source at the center, i.e. a solution of the equations (11) and (12) for $M_B = M_\odot$.

In the vicinity of the sun, the term $\delta\Phi_\nu(r)$ is much smaller than $\Phi_B(r)$. Near the center, the potential variable $v(x)$ has the asymptotic behaviour

$$v(x) \approx \frac{-4}{3}x^{3/2} + v'(0)x + 1, \quad (23)$$

where $v'(0)$ parametrizes solutions with different halo masses. Inserting the last expression into Eq. (22), we find for the additional potential energy due to the neutrinos

$$\delta U = m\delta\Phi_\nu(r) = \frac{4mGM_\odot}{3a_\nu^{3/2}}r^{1/2} + C, \quad (24)$$

where $C = v'(x_0) - v'(0)$ is a constant which is irrelevant for our further considerations, and m is the mass of the planet or asteroid. Thus our problem reduces to investigating the perihelion shifts of the planets and asteroids due to the small perturbation given by Eq. (24).

When a small correction $\delta U(r)$ is added to the potential energy $U = -\alpha/r$, the paths of finite motion are no longer closed, and at each revolution, the perihelion is displaced by a small angle (Landau and Lifshitz 1960)

$$\delta\varphi = \frac{\partial}{\partial L} \left(\frac{2m}{L} \int_0^\pi r^2 \delta U d\varphi \right). \quad (25)$$

Here L is the angular momentum of the planet or asteroid whose unperturbed orbit is given by the standard equation for an ellipse, i.e.

$$r = \frac{p}{1 + e \cos \varphi}, \quad (26)$$

with

$$p = a(1 - e^2), \quad e = 1 + \frac{2EL^2}{m\alpha^2}, \quad a = \frac{\alpha}{2E}, \quad \alpha = mGM_\odot, \quad L^2 = pm\alpha, \quad (27)$$

where E is the energy of the planet or asteroid ($E < 0$), e is the eccentricity, a the semi-major axis and p the latus rectum of its orbit.

Before calculating the perihelion shifts due to the neutrino halo, let us consider a general perturbation potential of the power law form

$$\delta U = \gamma r^n, \quad (28)$$

where γ and n are constant real numbers. For instance, it is well known that the general relativistic corrections for a planet moving around the sun can be incorporated in the Newtonian framework via an effective potential of the form

$$U_{eff} = -\frac{\alpha}{r} + \frac{L^2}{2mr^2} - \frac{\beta}{r^3}, \quad (29)$$

where the constant β is given by

$$\beta = \frac{GM_\odot L^2}{mc^2}. \quad (30)$$

The last term in the Eq. (29), is the general relativistic correction to the potential δU with $\gamma = \beta$ and $n = -3$. Due to this term, the perihelion is shifted by an angle $\delta\varphi_{GRT}$ which can be easily calculated from Eq. (25) yielding

$$\begin{aligned} \delta\varphi_{GRT} &= \frac{\partial}{\partial L} \left[\frac{2m}{L} \int_0^\pi r^2 \frac{-\beta}{r^3} d\varphi \right] = -2m^2\alpha\beta \frac{\partial}{\partial L} \left[\frac{1}{L^3} \int_0^\pi (1 + e \cos \varphi) d\varphi \right] \\ &= 6\pi m^2\beta\alpha L^{-4} = \frac{6\pi GM_\odot}{c^2 a(1 - e^2)} \end{aligned} \quad (31)$$

for the general relativistic perihelion shift per revolution. Let us assume that, in addition to the general relativistic perturbation potential $-\beta/r^3$, there is a further perturbative potential given by Eq. (28) due to

some type of DM. It is clear that this small shift will be additive in first order. The contribution to the shift per revolution due to the perturbation potential δU is thus

$$\delta\varphi = \frac{\partial}{\partial L} \left[\frac{2m}{L} \int_0^\pi \gamma r^n r^2 d\varphi \right] = 2m\gamma \frac{\partial}{\partial L} \left[\frac{1}{L} \int_0^\pi \frac{p^{n+2}}{(1 + e \cos \varphi)^{n+2}} d\varphi \right]. \quad (32)$$

After some algebraic manipulations, we arrive at the expression for the shifts per revolution

$$\delta\varphi = \frac{2\gamma}{\alpha} \left(a(1 - e^2) \right)^{n+1} \left[(2n + 3)I_{n+2}(e) + \frac{1 - e^2}{e^2} (n + 2) \left(I_{n+2}(e) - I_{n+3}(e) \right) \right], \quad (33)$$

where the function $I_n(e)$ has been defined as

$$I_n(e) = \int_0^\pi \frac{1}{(1 + e \cos \varphi)^n} d\varphi = \frac{\pi}{(1 - e^2)^{n/2}} P_{n-1} \left(\frac{1}{\sqrt{1 - e^2}} \right), \quad (34)$$

$P_{n-1}(x)$ being the Legendre polynomial of degree $n - 1$. In the limit of small eccentricities, i.e. $e \rightarrow 0$, Eq. (33) yields

$$\delta\varphi = -\frac{\gamma\pi}{\alpha} a^{n+1} n(n + 1). \quad (35)$$

The perihelion shifts become negative, i.e. $\delta\varphi < 0$ for $n > 0$ and $n < -1$ assuming of course $\gamma > 0$. In the following, we will deal with two special cases: degenerate neutrino DM and DM with a constant density which could describe homogeneous WIMP DM in the vicinity of the sun.

In the case of a neutrino halo, the perturbative potential δU is given by Eq. (24). We can thus apply Eq. (33), with $n = 1/2$ and $\gamma = 4mGM_\odot/(3a_\nu^{3/2})$. The constant C in Eq. (24) does not contribute to the perihelion shift. After integration, we arrive at the perihelion shift per revolution

$$\delta\varphi = -\frac{8}{3} \left(\frac{a}{a_\nu} \right)^{3/2} \frac{(e + 1)(1 - e)^{1/2}}{e^2} \left[E(\pi/2, k) + (e - 1)F(\pi/2, k) \right], \quad (36)$$

where k is given by

$$k = \sqrt{\frac{2e}{1 + e}}, \quad (37)$$

and $F(\pi/2, k)$ and $E(\pi/2, k)$ are the complete elliptic integrals of the first and second kind, respectively, defined as

$$F(\pi/2, k) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}}, \quad (38)$$

$$E(\pi/2, k) = \int_0^{\pi/2} d\varphi \sqrt{1 - k^2 \sin^2 \varphi}. \quad (39)$$

We can now write the expression for the perihelion shift per century as

$$\begin{aligned} \delta\tilde{\varphi} &= -5.50'' \times 10^5 \left(\frac{a_E}{a_\nu} \right)^{3/2} \frac{(e + 1)(1 - e)^{1/2}}{e^2} \\ &\times \left[E(\pi/2, k) + (e - 1)F(\pi/2, k) \right] \times \frac{100}{T_E}, \end{aligned} \quad (40)$$

where Kepler's third law has been used to eliminate the period T of the planet, i.e.

$$T = T_E \frac{a^{3/2}}{a_E^{3/2}}. \quad (41)$$

Here, T_E and a_E are the period and the semi-major axis of the Earth's orbit, respectively. From Eq. (40), we conclude that the shift does not depend on the semi-major axis a , i.e. they are a function of the eccentricity e only. Moreover, the shift is negative in contrast to the general relativistic one (see Eq. (31)). In Fig. 5, we present the shift due a neutrino halo as a function of the eccentricity for various neutrino masses. The data points show the difference between the observed perihelion shift (obs) and the correction due to the general relativity theory (GRT), i.e. $\delta\tilde{\varphi}_{obs} - \delta\tilde{\varphi}_{GRT}$. Thus, if there was no neutrino halo, $\delta\tilde{\varphi}$ should vanish identically. The general relativistic corrections, calculated using the exact general relativistic equations rather than the useful approximation eq. (40), as well as the observed values are taken from Weinberg (1972). The data points with error bars are shown for Venus, Earth, Mercury and Icarus. As expected, the error bars of the data points decrease for increasing eccentricity. We may thus obtain a good upper bound for the neutrino mass using the perihelion shift data for Icarus. In Fig. 6, the expected neutrino halo shift is plotted for Icarus as a function of the neutrino mass. The horizontal lines denote the region of $\delta\tilde{\varphi} = \delta\tilde{\varphi}_{obs} - \delta\tilde{\varphi}_{GRT}$ allowed by observations. The observational shifts for Icarus is $\delta\tilde{\varphi}_{obs} = 9.8'' \pm 0.8''$ per century, while the general relativistic correction is $\delta\tilde{\varphi}_{GRT} = 10.3''$ per century (Weinberg 1972). Requiring that the shift due to a possible neutrino halo cannot be smaller than the lower limit of $\delta\tilde{\varphi} = \delta\tilde{\varphi}_{obs} - \delta\tilde{\varphi}_{GRT} = -0.5'' \pm 0.8''$, i.e. $\delta\tilde{\varphi} \gtrsim -1.3''$ per century, we obtain an upper limit for the neutrino mass of

$$\begin{aligned} m_\nu c^2 &\leq 16.4 \text{ keV for } g_\nu = 2, \\ m_\nu c^2 &\leq 13.8 \text{ keV for } g_\nu = 4, \end{aligned} \quad (42)$$

as seen from Fig. 6. Here, we note that the bounds (42) agree very well with those obtained from the mass excesses in Figs. 1 and 2.

Moreover, considering the violent supermassive dark object at the center of M87, with mass $M = (3.2 \pm 0.9) \times 10^9 M_\odot$ (Macchetto et al. 1997), as a relativistic neutrino ball at the Oppenheimer-Volkoff limit (Bilić, Munyaneza and Viollier 1999), the neutrino mass is constrained by

$$\begin{aligned} m_\nu c^2 &\leq 16.5 \text{ keV for } g_\nu = 2 \\ m_\nu c^2 &\leq 13.9 \text{ keV for } g_\nu = 4. \end{aligned} \quad (43)$$

Such a neutrino ball would be virtually indistinguishable from a supermassive black hole, as its radius is 4.45 Schwarzschild radii, little more than the radius of the last stable orbit around a black hole of three Schwarzschild radii. In fact, as the mass density is very small near the surface of the neutrino ball, the "effective" neutrino ball radius is substantially smaller than 4.45 Schwarzschild radii. Furthermore, if we interpret the supermassive dark object of mass $M = (2.6 \pm 0.2) \times 10^6 M_\odot$ at the Galactic center (Ghez et al. 1998) as a neutrino ball (Viollier, Trautmann and Tupper 1993), the observed motion of stars close to Sgr A* yields an upper limit for the radius of the compact dark object, and therefore a lower limit for the neutrino mass (Munyaneza, Tsiklauri and Viollier 1998, 1999). Finally, the infrared drop of the emission spectrum of Sgr A*, interpreted in terms of standard thin accretion disk theory, provides us with an upper limit of the neutrino mass (Bilić, Tsiklauri and Viollier 1998, Tsiklauri and Viollier 1999; Munyaneza and Viollier 1999), because of the cutoff of the emission of disk radiation inside the neutrino ball. Combining these two constraints, we obtain

$$\begin{aligned} 15.9 \text{ keV} \leq m_\nu c^2 &\leq 25 \text{ keV for } g_\nu = 2, \\ 13.4 \text{ keV} \leq m_\nu c^2 &\leq 21 \text{ keV for } g_\nu = 4. \end{aligned} \quad (44)$$

Such a neutrino ball would differ substantially from a supermassive black hole, as the escape velocity from the center would be only about $v_\infty \approx 1700 \text{ km/s}$ and the radius is about 4×10^4 Schwarzschild radii.

Virtually all supermassive compact dark objects that have been observed so far at the centers of galaxies have masses in the range of $10^{6.5}$ to $10^{9.5}M_\odot$ and could therefore be explained in the neutrino ball scenario.

We now turn to the interpretation of DM in the solar system at constant density (Grøn and Soleng 1996) which could describe WIMP DM that is not clustered around the sun. In this case, we have $n = 2$ and $\gamma = 2\pi m G \rho_d / 3$, where ρ_d is the density of DM in the solar system. Eq. (33) yields for the shifts per century

$$\delta\tilde{\varphi} = -\frac{4\pi^2 \rho_d a^3}{M_\odot} (1 - e^2)^{1/2} \times \frac{100}{T}. \quad (45)$$

Here T is the Icarus period and a its semi-major axis. From Eq. (45), we gather that the shifts are negative, as in the previous case, but they now depend on two parameters: the semi-major axis a and the eccentricity e . We can establish an upper limit on the density of DM using the Icarus data. Its eccentricity is $e = 0.827$, the semi-major axis $a = 1.076$ AU and the period $T = 1.116$ yr. By requiring that the DM shift is restricted to $\delta\tilde{\varphi} \gtrsim -1.3''$ per century, we get an upper bound on the DM density in the solar system of

$$\rho_d \lesssim 1.5 \times 10^{-15} \text{ g/cm}^3. \quad (46)$$

Such a constant density DM model was studied by Grøn and Soleng (1996) in the framework of general relativity. However, they derived an upper limit of the DM density of $\rho_d \lesssim 1.8 \times 10^{-16} \text{ g/cm}^3$ from the perihelion motion of Icarus. If we use $-0.8''$ instead of $-1.3''$ for the lower limit of the DM shift of Icarus, as Grøn and Soleng (1996) did, we obtain a density of $\rho_d \lesssim 9.4 \times 10^{-16} \text{ g/cm}^3$ which is still larger by a factor of 5.2 than the upper limit obtained by Grøn and Soleng (1996). The discrepancy between our results and those by Grøn and Soleng can be traced back to the fact that those authors used for their calculations a perturbation expansion in terms of the eccentricity valid only for nearly circular orbits ($e \approx 0$) while Icarus has an eccentricity of $e = 0.827$. Using our value of the upper limit for ρ_d , i.e. Eq. (46), one can calculate the total DM mass within the orbits of the various planets. Within the Earth's orbit, the DM mass is $M_d \leq 1.1 \times 10^{-8} M_\odot$ which is in agreement with the bound of $M_d \lesssim 7 \times 10^{-8} M_\odot$ obtained by Mikkelsen and Newman (1977). Within Uranus' orbit, we get a DM mass bound $M_d \leq 7.6 \times 10^{-5} M_\odot$, and, finally for the DM mass within Neptune's orbit we have $M_d \leq 2.9 \times 10^{-4} M_\odot$. The last two bounds are clearly in conflict with the observed ephemeris, which allow only a DM mass of the order of a few times $10^{-6} M_\odot$, even within the orbit of Neptune (Anderson et al. 1995). We thus conclude that if the DM density is constant, the upper bound of DM within Neptune's orbit restricts the DM density in the solar system to $\rho_d \lesssim 1.5 \times 10^{-17} \text{ g/cm}^3$.

4. Other observational consequences of a degenerate neutrino halo

We now turn to the question whether a neutrino halo, with properties as described in the last sections, can be observed in nature, using other than gravitational detection techniques. Let us consider the most conservative scenario, in which the Standard Model of Particle Physics (SMP), minimally modified to accommodate three species of massive neutrinos, that are mixed through a leptonic Cabibbo-Kobayashi-Maskawa (CKM) matrix, is basically correct at low energies. We further assume that our heavy neutrino is a Dirac neutrino, more specifically the mass eigenstate ν_τ , in the mass range between $10 \text{ keV}/c^2$ and $25 \text{ keV}/c^2$, while the ν_e and ν_μ are assumed to be massless. The ν_τ couples preferentially to the ν'_τ and to a lesser extent to the ν'_e and ν'_μ eigenstates of the charged weak interaction. In the framework of the SMP, the dominant decay mode of the mass eigenstate ν_τ in the assumed mass range is the conventional radiative

decay, having a lifetime (Boehm and Vogel 1987) of

$$\tau_D(\nu_\tau \rightarrow \nu_i \gamma) = 1.30 \times 10^{15} \left(\frac{17.2 \text{ keV}}{m_\nu c^2} \right)^5 |U_{\tau\nu_\tau} U_{\tau\nu_i}^*|^{-2} \text{ yr.} \quad (47)$$

Here the CKM- matrix element $U_{\tau\nu_i}$ ($i = e, \mu$) with the largest modulus, presumably $i = \mu$, determines the dominant decay mode. The present limit from the CHORUS collaboration for $|U_{\tau\nu_\mu}|^2$ is $|U_{\tau\nu_\mu}|^2 < 3.3 \times 10^{-4}$ for $\delta m_{\tau\mu}^2 \gtrsim 100 \text{ eV}^2$ (Sato 1999). One can thus safely conclude that the ν_τ is quasistable over the lifetime of the Universe. However, even though the τ -neutrino is remarkably stable against radiative decay, τ -neutrino matter is quite radioactive. It is perhaps so abundant that the X-ray flux generated by the τ -neutrino decay rates

$$\dot{n}_\nu = -\frac{1}{\tau_D} n_\nu, \quad (48)$$

where the neutrino (and antineutrino) number density n_ν is obtained from Eq. (13)

$$n_\nu(r) = \frac{\rho_\nu(r)}{m_\nu} = \frac{8m_\nu^7 g_\nu^2 G^3 M_\odot^2}{9\pi^3 \hbar^6} \left(\frac{v}{x} \right)^{3/2}, \quad (49)$$

could be observable. In the vicinity of a pointlike baryonic star of mass M_B , i.e. $v \approx M_B/M_\odot$, may be rewritten as

$$n_\nu \approx \frac{m_\nu^3 g_\nu}{6\pi^2 \hbar^3} \left(\frac{2GM_B}{r} \right)^{3/2} = 0.978 \times 10^{17} g_\nu \left(\frac{M_B R_\odot}{M_\odot r} \right)^{3/2} \left(\frac{m_\nu c^2}{17.2 \text{ keV}} \right)^3 \text{ cm}^{-3}. \quad (50)$$

Thus the number of photons with energy $m_\nu c^2/2$ emitted per unit time and volume is

$$\dot{n}_\nu = -75.2 g_\nu \left(\frac{M_B R_\odot}{M_\odot r} \right)^{3/2} \left(\frac{m_\nu c^2}{17.2 \text{ keV}} \right)^8 |U_{\tau\nu_\tau} U_{\tau\nu_i}^*|^2 \text{ cm}^{-3} \text{ yr}^{-1}, \quad (51)$$

Although an energy of $m_\nu c^2/2 \approx 8 \text{ keV}$ is equivalent to a temperature of roughly 10^8 K , the X-ray flux from neutrino decays near the solar surface is too small to contribute significantly to nuclear synthesis in the sun or to maintaining the solar corona at a temperature of a few million degrees. At a distance of one astronomical unit ($r = a_E$), the number density and decay rates per unit volume are

$$n_\nu \approx 3.10 \times 10^{13} g_\nu \left(\frac{m_\nu c^2}{17.2 \text{ keV}} \right)^3 \text{ cm}^{-3}, \quad (52)$$

and

$$\dot{n}_\nu \approx -2.38 \times 10^{-2} g_\nu \left(\frac{m_\nu c^2}{17.2 \text{ keV}} \right)^8 |U_{\tau\nu_\tau} U_{\tau\nu_\mu}^*|^2 \text{ cm}^{-3} \text{ yr}^{-1}, \quad (53)$$

Thus, if our model of the solar neutrino halo is correct, we predict that in a shielded vacuum of 1000 m^3 , one will observe g_ν photons per hour for $|U_{\tau\nu_\tau} U_{\tau\nu_\mu}^*|^2 = 3.3 \times 10^{-4}$ which is the present experimental limit of the CHORUS collaboration, and $m_\nu = 16 \text{ keV}/c^2$, g_ν being the spin degeneracy factor of neutrinos (Viollier, Leimgruber and Trautmann 1992). However, this number could be substantially enhanced through the gravitational field of the Earth. The photons originating from the radiative decay $\nu_\tau \rightarrow \nu_\mu + \gamma$ or ($\nu_\tau \rightarrow \nu_e + \gamma$) will have a sharp energy of $m_\nu c^2/2$ with $\Delta E/E \approx 10^{-4}$. This measurement could prove the existence of the massive neutrino halo and fix the ν_τ mass and the $\nu_\tau - \nu_\mu$ (or $\nu_\tau - \nu_e$ mixing angle accurately. At the same time, this could be the first direct evidence for the neutrino background and the nature of DM.

If both neutrinos and antineutrinos are present in a neutrino halo around the sun, these will annihilate into light neutrinos $\nu_e, \bar{\nu}_e$ and $\nu_\mu, \bar{\nu}_\mu$ through ordinary weak interactions processes via the Z^0 , which are independent of the mixing angle of course. In fact, the rate of change of the neutrino (and antineutrino) number density is

$$\dot{n}_\nu = - \langle \sigma_A v_\nu \rangle \frac{n_\nu^2}{2}, \quad (54)$$

where the spin averaged annihilation cross section is for Dirac neutrinos given by

$$\langle \sigma_A v_\nu \rangle = \frac{G_F^2 m_\nu^2 c}{\pi \hbar^4 g_\nu}. \quad (55)$$

The largest annihilation rate is obtained in the interior of the star which of course depends on the internal structure of the star. However, the effective annihilation time τ_A at the surface of the star with mass M_B and radius R_B is a good indicator for how fast this process actually is. It can be calculated quite reliably yielding

$$\begin{aligned} \tau_A &= \left| \frac{n_\nu}{\dot{n}_\nu} \right| = \frac{2}{\langle \sigma_A v_\nu \rangle n_\nu} \approx \frac{12\pi^3 \hbar^7}{G_F^2 m_\nu^5 c} \left(\frac{R_B}{2GM_B} \right)^{3/2} \\ &= 0.4335 \times 10^{13} \left(\frac{17.2 \text{ keV}}{m_\nu c^2} \right)^5 \left(\frac{R_\odot M_\odot}{R_B M_B} \right)^{3/2} \text{ yr}. \end{aligned} \quad (56)$$

Thus for a neutrino mass of $m_\nu = 16 \text{ keV}/c^2$ and a solar mass $M_B = M_\odot$ with a radius $R_B = R_\odot$, we obtain $\tau_A = 0.623 \times 10^{13} \text{ yr}$, much larger than the age of the universe. Although this annihilation process is more efficient than the radiative decay, it will be essentially unobservable due to the low energy of the neutrinos.

5. Summary

In this paper, we have investigated the properties and implications of a possible halo of degenerate neutrino matter around the sun. For small halo masses or sufficiently close to the center, this neutrino halo is dominated by the gravitational potential of the sun. We have established that the enclosed mass of a degenerate neutrino halo around the sun is, for neutrino masses of $m_\nu \sim 15 \text{ keV}/c^2$, of the order of a few times $10^{-6} M_\odot$ within Uranus' and Neptune's orbits, consistent with available observational data. If such a neutrino halo exists, it would decrease the perihelion shifts, i.e. the neutrino halo shifts are negative in contrast to the general relativistic ones. The perihelion shifts due to the neutrino halo depend only on one parameter, the eccentricity e , while those due to general relativistic effects depend on the eccentricity e and the semi-major axis a . The maximal radius of such a degenerate neutrino halo around the sun is a few light years, with a total halo mass of $\sim 3 M_\odot$.

In order to explain the mass excesses within the orbits of various outer planets using the Voyager 1/2 and Pioneer 10/11 data, the neutrino mass should be in a narrow range of $m_\nu = (15 \pm 1) \text{ keV}/c^2$ for $g_\nu = 2$. The predicted values of the acceleration in the solar system have been compared to those obtained from the recent observations. We have seen that a degenerate heavy neutrino halo around the sun fits the recent ephemeris very well, but it cannot explain the anomalous acceleration in the Pioneer data (Anderson et al. 1998; Turyshev 1999). We have shown that, in order to be consistent with the observations, the total mass of the neutrino halo should be less than $\sim 10^2 M_\odot$. A neutrino mass in the range from 14 to 16 keV/c^2 fits all the observational data on DM in the solar system, the compact dark central object Sgr A* in our

Galaxy with $M = (2.6 \pm 0.2) \times 10^6 M_\odot$, and the most massive compact dark object at the center of the galaxy M87 with $M = (3.2 \pm 0.9) \times 10^9 M_\odot$.

We have established an upper bound of DM with a constant density around the sun of $\rho_d \lesssim 1.5 \times 10^{-17} \text{g/cm}^3$ using the upper limit for the DM mass within Neptune’s orbit.

We have proposed a new experiment aimed at observing the radiative decay of the neutrinos in the mass range around $15 \text{ keV}/c^2$, which the sun might have accumulated in a degenerate neutrino halo during its formation.

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Figure captions:

Fig1: Mass $\mathcal{M}_\nu(r)$ of degenerate neutrino matter enclosed within a radius r from the sun for various neutrino masses. The total mass of the halo is $0.7M_\odot$. The observational data points with error bars (Anderson et al. 1995) are shown for Jupiter ($a = 5.2$ AU), Uranus ($a = 19.2$ AU) and Neptune ($a = 30.1$ AU). The data point at $r = 1$ AU is taken from Mikkelsen and Newman (1977). The arrows indicate upper limits. The dark mass needed to explain the anomalous acceleration (Anderson et al. 1998; Turyshev et al. 1999) is indicated by a bar at 40 to 60 AU.

Fig2: Mass $\mathcal{M}_\nu(r)$ of degenerate neutrino matter enclosed within a radius r from the sun for various total masses M_ν of the neutrino halo. The neutrino mass and degeneracy factor are fixed to $m_\nu = 14$ keV/ c^2 and $g_\nu = 2$, respectively. A total mass of the neutrino halo which is less than $M_\nu \sim 100M_\odot$ would be consistent with the observed mass excess data from Pioneer 10 and 11 (Anderson et al. 1995). The data points with error bars are shown for Jupiter, Uranus and Neptune. The observational data at $r = 1$ AU is taken from Mikkelsen and Newman (1977). The bar at 40 to 60 AU represents the dark mass needed to explain the anomalous acceleration of $a_P = 7.5 \times 10^{-8}$ cm/s².

Fig3: Excess acceleration a_ν due to a neutrino halo around the sun. The total mass of the halo is $M_\nu = 0.7 M_\odot$. The expected values of the acceleration using observational data points (Anderson et al. 1995) are shown by points with arrows. The point with arrow at $r = 1$ AU is calculated using the data from Mikkelsen and Newman (1977). The anomalous acceleration at 40 to 60 AU is indicated by a horizontal bar.

Fig4: The total mass M_ν as a function of the radius R_0 of the neutrino halo around the sun. The neutrino mass m_ν is varied as shown on the graph. $M_{\nu,max}$ turns out to be fairly constant, i.e. $M_{\nu,max} \approx 3M_\odot$, for the maximal radius R_{max} of the neutrino halo.

Fig5: Perihelion shifts $\delta\tilde{\varphi}$ caused by a neutrino halo as a function of the eccentricity e . The data points with error bars denote the difference between the observed perihelion shifts and general relativistic corrections for the perihelion shifts. The data points shown are for Venus ($e = 0.007$), Earth ($e = 0.017$), Mercury ($e = 0.206$) and Icarus ($e = 0.827$).

Fig6: The Icarus perihelion shift $\delta\tilde{\varphi}$ as a function of the neutrino mass m_ν . The horizontal lines show the difference between the observed value and the correction predicted by general relativity theory. In order to be consistent with the observational data for the Icarus perihelion shift, the neutrino mass is constrained by $m_\nu c^2 \leq 16.4$ keV for $g_\nu = 2$ or $m_\nu c^2 \leq 13.8$ keV for $g_\nu = 4$.

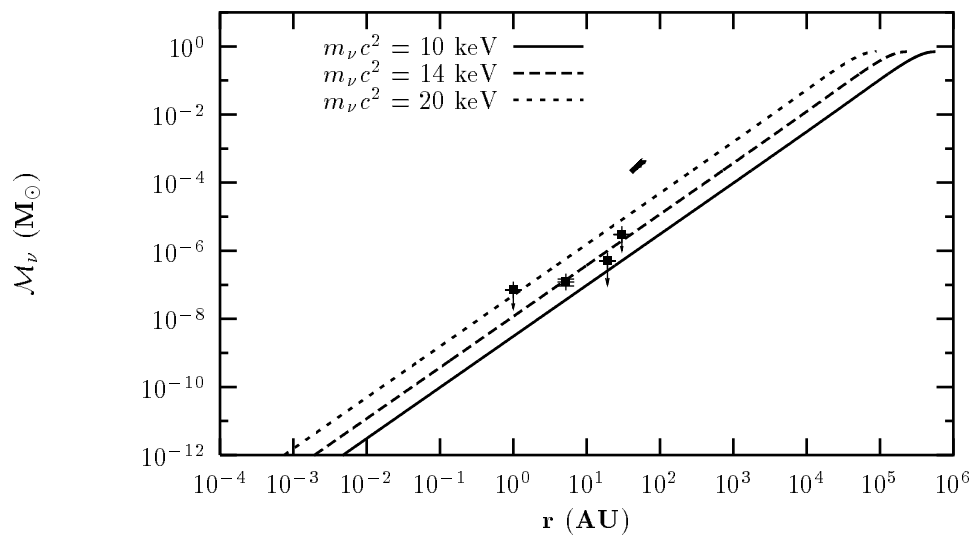


Fig. 1

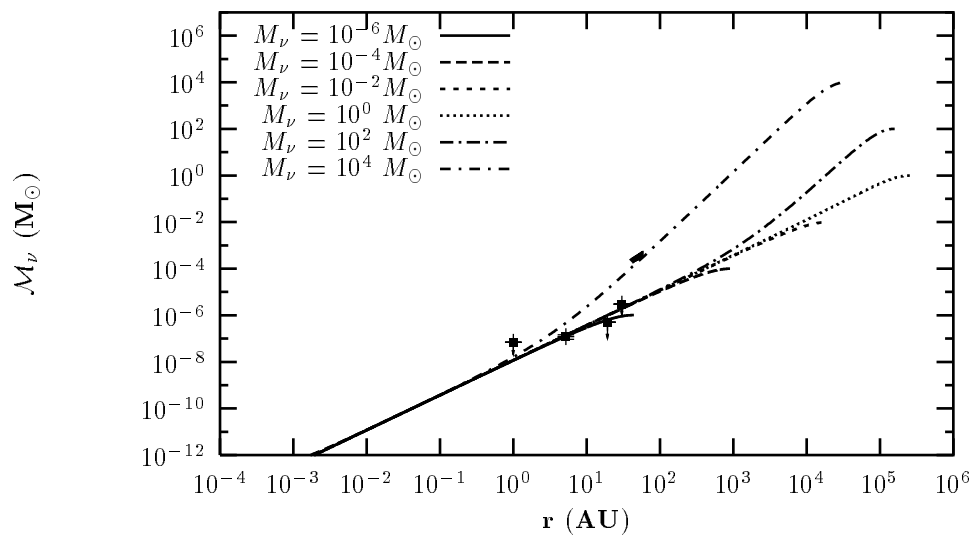


Fig. 2

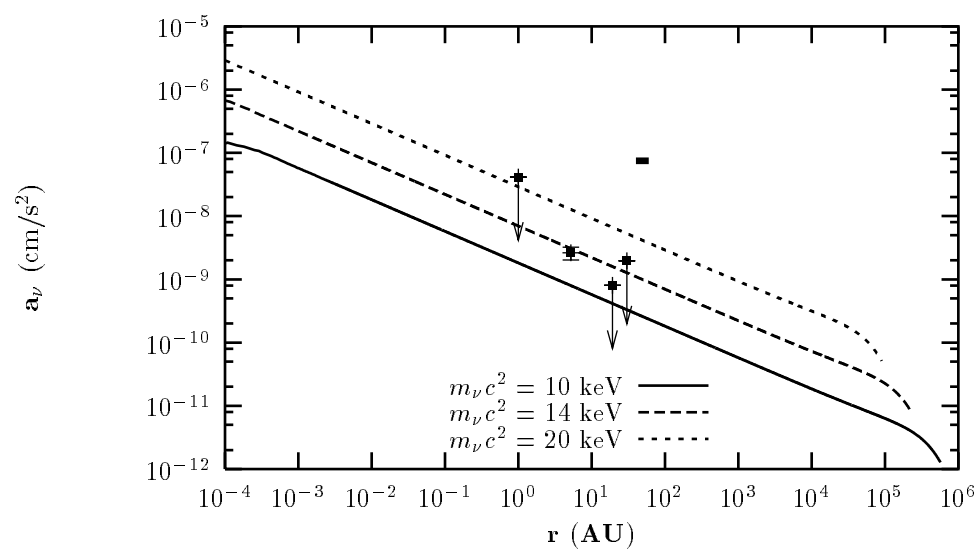


Fig. 3

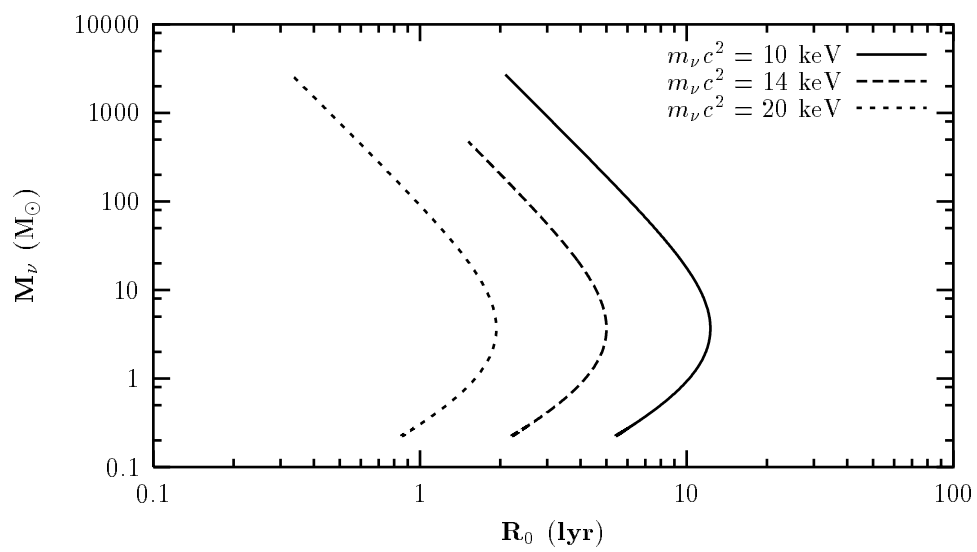


Fig. 4

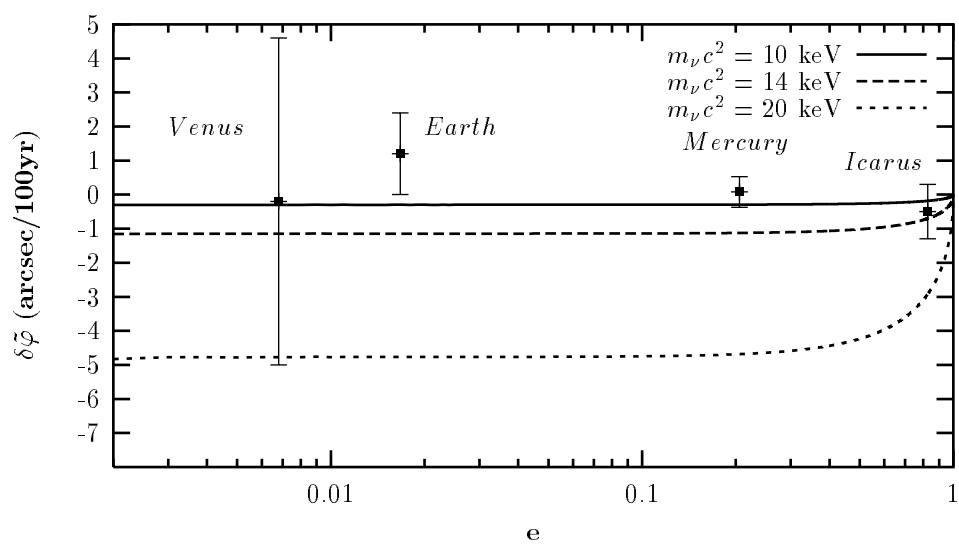


Fig. 5

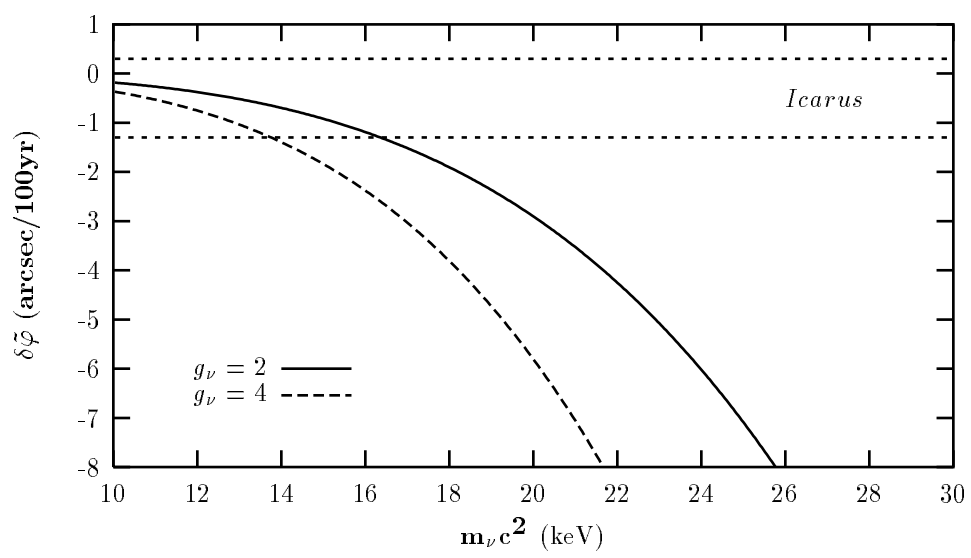


Fig. 6